Appendix A

Correction procedure for finite telescope opening angles

In order to construct flux maps, count rates have to be converted to physical units (fluxes) and averaged over two dimensional coordinate grids. The conversion to fluxes is an iterative process because of the finite aperture of the detectors, i.e. the true unidirectional flux has to be derived from the measured count rate in successive approximations.

A.1 Conversion to fluxes

The description of geometric factor and directional response in this section is based on a paper by Sullivan (1971).

A.1.1 General formulation

The coincidence counting rate of any particle telescope depends upon the effective dimensions and positions of the telescope sensors as well as on the sensor efficiencies. For an ideal telescope—whose efficiency for detecting particles of a given type is one in a given energy interval and zero otherwise and whose sensors are mathematical surfaces with no thickness—the factor of proportionality relating the counting rate C to the integral directional particle flux J is defined as the gathering power Γ of the telescope. When the flux is isotropic, i.e. $J = J_0$, the factor of proportionality is called the geometric factor G:

$$C = G\, J_0 \,. \tag{A.1}$$

Exact expressions can be obtained for the geometric factor and directional response of cylindrically symmetric telescopes.

The coincidence counting rate of a particle telescope can be expressed as:

$$C(\mathbf{x}, t_0) = \frac{1}{T} \int_{t_0}^{t_0+T} dt \int_S \mathbf{r} \cdot d\sigma \int_\Omega d\omega \int_0^\infty dE \;\; \sum_i \varepsilon_i(E, \sigma, \omega, t)\, j_i(E, \omega, t)\,, \tag{A.2}$$

where

C	=	coincidence counting rate (s^{-1}),
$\mathbf{x}$	=	position vector of the telescope
i	=	label for type of particle,
j_i	=	differential directional flux of particle type i ($\mathrm{s}^{-1}\mathrm{cm}^{-2}\mathrm{sr}^{-1}\mathrm{MeV}^{-1}$),
ε_i	=	detection efficiency for particle type i,
t	=	time,
t_0	=	time at start of observation,
T	=	total observation time,
$d\sigma$	=	element of surface area of the last sensor to be penetrated,
S	=	total area of the last telescope sensor,
$\mathbf{r}$	=	unit vector specified by spherical coordinates (θ, ϕ),
$d\omega = -d\phi\, d\cos\theta$	=	element of solid angle around $\mathbf{r}$,
Ω	=	domain of Ω defined by the other telescope sensors,
$\mathbf{r}\cdot d\sigma$	=	effective element of area looking into ω.

Equation (A.2) expresses the requirements for the detection of a particle. Although it is quite general, the following implicit assumptions have been made:

1. $d\sigma$, ω, and $\mathbf{x}$ are time independent;
2. no transformation of particle type occurs other than that included in ε_i;
3. the particle trajectory is a straight line.

Dropping these assumptions severely complicates the treatment of the problem and renders an analytic solution difficult. The first assumption may not be valid for a rapidly spinning satellite and/or long accumulation times.

To simplify the problem further, we consider only ideal telescopes where the efficiency is independent of ω, σ and t, and consider only one particle type (henceforth, we will drop the subscript denoting particle type).

With the assumption that j is independent of t and separates into

$$j(E,\omega) = j_0(E)\, F(\omega)\,, \tag{A.3}$$

where $F(\omega)$ is normalised so that $\int F(\omega)\, d\omega = 1$, Eq. (A.2) becomes

$$C = \left[\int_\Omega d\omega \int_S F(\omega)\, \mathbf{r}\cdot d\sigma\right] J \equiv \Gamma_F\, J\,, \tag{A.4}$$

where

$$J = \int_0^\infty j_0(E)\, \varepsilon(E)\, dE\,. \tag{A.5}$$

In the case of a detector with well defined energy channels with uniform response

$$\varepsilon_1 = \begin{cases} 1, & E_\mathrm{l} \le E \le E_\mathrm{u}\,, \\ 0, & E < E_\mathrm{l},\; E > E_\mathrm{u}\,, \end{cases} \tag{A.6}$$

J is given by

$$J = \int_{E_\mathrm{l}}^{E_\mathrm{u}} j_0(E)\, dE\,, \tag{A.7}$$

which, for small energy ranges, can be approximated by

$$J = j_0[(E_\mathrm{l} + E_\mathrm{u})/2]\,(E_\mathrm{u} - E_\mathrm{l})\,. \tag{A.8}$$

The expression in square brackets in Eq. (A.4) is the gathering power Γ_F of the telescope when the intensity has an angular dependence given by $F(\omega)$, i.e.

$$\Gamma_F = \int_\Omega d\omega \int_S F(\omega)\,\mathbf{r}\cdot d\sigma = \int_\Omega F(\omega)\,d\omega \int_S \mathbf{r}\cdot d\sigma\,. \tag{A.9}$$

The directional response function $R(\omega)$ of a telescope can be defined as:

$$R(\omega) = \int_S \mathbf{r}\cdot d\sigma\,. \tag{A.10}$$

For a telescope with cylindrical symmetry the effective area h is related to R as:

$$h(\theta)\cos\theta = \int_S \mathbf{r}\cdot d\sigma\,. \tag{A.11}$$

With this definition Eq. (A.9) can be rewritten as

$$\Gamma_F = \int_0^{2\pi}\int_0^{\theta_1} F(\theta,\phi)\,h(\theta)\,\cos\theta\,\sin\theta\,d\theta\,d\phi\,, \tag{A.12}$$

where θ_l is the telescope opening half angle. If the flux is isotropic then F is unity and the geometric factor (the gathering power for isotropic flux) depends only on the geometry of the telescope, i.e.:

$$G = \Gamma_1 = 2\pi \int_0^{\theta_1} h(\theta)\,\cos\theta\,\sin\theta\,d\theta\,. \tag{A.13}$$

A.1.2 Single element telescope

For an ideal telescope consisting of a single planar detector without shielding, $h(\theta) = A$ with A the surface area of the detector, so that the geometric factor is given by

$$G = 2\pi\,A \int_0^1 (-\cos\theta)\,d(\cos\theta) = \pi A\,. \tag{A.14}$$

If particles are incident from both sides then the detector area is doubled, and

$$G = 2\pi\,A\,. \tag{A.15}$$

The gathering power and effective area are also easily evaluated from Eqs. (A.9) and (A.11). A single detector embedded in a viewing cone with opening angle smaller than π can be treated as the lower detector of a two-element telescope in which the respective surfaces of both detectors and their separation define the same solid angle as the viewing cone of the single detector.

A.1.3 Multi-element telescope

For a multi-element telescope with cylindrical symmetry, the effective area can be written in analytical form, although the derivation becomes tedious for more than two detectors. The gathering power and geometric factor can be determined by integration, which may involve elliptical integrals depending on the form of the angular dependence F of the intensity.

For complex geometries a numerical approximation usually is easier than the analytical approach. This technique involves numerical integration of the effective area taking into account the path of an incoming particle through a mathematical description of the detector plates.

A.2 Geometric factor correction

The quantity typically measured by a particle telescope is the number of incoming particles N over the accumulation period T, in the solid angle Ω defined by the telescope configuration and centered around a direction **r**, in the energy interval $[E_\mathrm{l}, E_\mathrm{u}]$ defined by the detector response. The physical quantity that the telescope aims to measure is the differential directional particle flux j.

In general, the trapped particle flux measured by a telescope differs from the true flux because of the finite opening angles of these instruments. A zero-order approximation of the true flux is given by:

$$j^{(0)}(E) = \frac{1}{G}\,\frac{N}{T}\,\frac{1}{E_\mathrm{u} - E_\mathrm{l}}\,, \tag{A.16}$$

where G is the nominal geometric factor of the detector element and E represents the reference value of the energy interval $[E_\mathrm{l}, E_\mathrm{u}]$.

The measured flux can be corrected by an iterative procedure:

1. The first step consists of averaging the zero-order fluxes given by Eq. (A.16) over an (E, L, α_0) grid, using the averages of the uncorrected measured flux as the zero-order approximation $j^{(0)}$. This is equivalent to assuming that the ambient flux is isotropic.

2. For step i, evaluate the gathering power Γ_F [Eq. (A.12)] for each measurement (and for each energy channel), using the pitch angle dependence of the last iteration $j^{(i-1)}$.

3. For each measurement (and energy), integrate the zero-order flux (using the bin averaged $j^{(0)}$) over the telescope opening angle using Eqs. (A.12) and (A.4); compute the ratio of this integrated flux defined to $j^{(0)}$ corresponding to the (E, L, α_0) value for each measurement, and multiply the measured flux by this ratio.

4. Re-bin the measured fluxes, applying the correction factor from step 3. The new bin averages constitute the i^{th} order approximation $j^{(i)}$ of the true flux.

5. Repeat from step 2 until convergence is reached. One step should be sufficient.

This procedure ignores the dependence of the flux on azimuth. This approximation is reasonable when the measurements used for the flux averages were taken over the full azimuth range $[0°, 360°]$. Taking into account both pitch angle and azimuth dependence would significantly increase the complexity of the procedure, which is already very demanding in calculation time.

The gathering power [Eq. (A.12)] is determined by integrating the effective area function h over the telescope opening, whereby the flux dependence on α_0 of the previous iteration is used for $F(\theta)$ (we have ignored the dependence of the flux on ϕ). The integration is carried out in the variable θ, the off-axis angle, and the azimuthal angle ϕ measured in the plane perpendicular to the telescope axis. For a measurement point P and a local pitch angle α (corresponding to the pitch angle of the detector axis), the drift shell coordinates (B_m, L) can be evaluated with BLXTRA. The corresponding equatorial pitch angle α_0 is given by

$$\alpha_0 = \arcsin\left(\sqrt{\frac{B_0}{B_\mathrm{m}}}\right) , \tag{A.17}$$

with

$$B_0 \equiv \frac{0.311653}{L^3} . \tag{A.18}$$

The flux seen by the detector then is

$$j(E, L, \alpha_0) = \int_0^{2\pi} d\phi \int_0^{\theta_1} j^{(0)}[E, L, \alpha'(\theta, \phi)]\, h(\theta)\, \cos\theta\, \sin\theta\, d\theta\, , \tag{A.19}$$

where α' is the local pitch angle corresponding to the off-axis angle θ and the azimuthal angle ϕ. $\alpha'(\theta, \phi)$ Is given by:

$$\cos\alpha' = \cos\alpha\cos\theta - \sin\alpha\sin\theta\cos\phi\, . \tag{A.20}$$